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Instantons and non-renormalisation in AdS/CFT

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Abstract: The series of perturbative fluctuations around a multi-instanton contribution to a specific class of correlation functions of supercurrents in $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory is examined in the light of the AdS/CFT correspondence. Subject to certain plausible assumptions, we argue that a given term in the $1/N$ expansion in such a background receives only a finite number of perturbative corrections in the 't Hooft limit. Such instanton non-renormalisation theorems would explain, for example, the exact agreement of certain weak coupling Yang-Mills instanton calculations with the strong coupling predictions arising from D-instanton effects in string theory amplitudes. These non-renormalisation theorems essentially follow from the assumption of a well defined derivative (α') expansion in the string theory dual of the Yang-Mills theory.

Keywords: Nonperturbative Effects, 1/N Expansion, D-branes, String Duality.

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Contents

1. Introduction

The conjectured equivalence of type-IIB superstring theory on $AdS_5 \times S^5$ to the boundary $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills conformal field theory [1, 2, 3] has been tested by a variety of calculations at leading order in the large-N limit and at large values of the 't Hooft coupling, $\lambda = g^2_{_{YM}} N/4\pi$ ($g^2_{_{YM}}$ is the Yang-Mills coupling constant). Many of these tests, such as those of certain two and three point correlation functions, have relied on non-renormalisation theorems/conjectures [4] and therefore allow the meaningful comparison of the regimes of strong and weak 't Hooft coupling.

Clearly, the ideal way of developing the AdS/CFT correspondence beyond the limited large λ region in which it has so far been studied would be to explicitly quantize IIB superstring theory in an $AdS_5 \times S^5$ background. Unfortunately, this is a daunting problem, even at tree level — in part because of the presence of a nonzero condensate of $R \otimes R$ background fields associated with the nonzero F_5 flux. In the absence of an explicit construction of string amplitudes most concrete calculations have made use of known low order terms in the expansion of the effective supergravity action in powers of the dimensionless parameter α'/L^2 (L is the size of the AdS_5 and S^5 background and $\alpha'^{1/2}$ is the string distance scale). In fact, knowing the complete effective action for the massless fields of string theory would be sufficient to compute the Yang-Mills correlation functions of the relevant dual operators, but we are far from achieving this.

Nevertheless, in the following we will show how some reasonable assumptions concerning the structure of the low energy expansion of type-IIB string theory lead to a number of non-renormalisation theorems in the instanton sector of certain Yang-Mills correlation functions. To be concrete, we will consider the D-instanton contributions to the four graviton scattering amplitude and arrive at statements regarding the corresponding Yang-Mills instanton terms in the AdS/CFT dual correlation functions of four energy-momentum tensors. Similar statements also apply to any of the Yang-Mills correlations functions that are related by supersymmetry. Specifically, what we will see is that, for certain 'protected' parts of the correlation functions, the 't Hooft expansion around an instanton background has only a finite number of perturbative terms in λ at each order in $1/N$. In particular, we will see that at leading order in N only the λ-independent semi-classical term arises. This would account for the precise agreement (for any instanton number K, at leading order in N) between the D-instanton contributions, at leading order in the α' (or $1/\lambda$) expansion [5], with the semi-classical (small-λ) contributions of Yang-Mills instantons [6, 7]. In the case of two and three point functions the space-time dependence is completely determined by (super) conformal invariance. However, the matching of the string theory D-instanton and Yang-Mills instanton contributions to the protected correlation functions involves matching non-trivial functions of the space-time positions (functions of two independent cross ratios in the case of the correlator of four stress tensors), together with specific dependence on N, λ and the instanton number.

1.1 Overview of the correspondence

The AdS/CFT conjecture [1] gives a relation between the parameters of the string theory — the dimensionless $AdS_5 \times S^5$ scale L^2/α', the $R\otimes R$ scalar field, $C^{(0)}$, and the coupling constant $g = e^{\phi} = \tau_2^{-1}$ — and those of the Yang-Mills theory with gauge group SU(N).

$$g = \frac{g_{_{YM}}^2}{4\pi}\,, \qquad 2\pi C^{(0)} = \theta\,, \qquad \frac{L^4}{\alpha'^2} = g_{_{YM}}^2 N \equiv 4\pi\lambda\,, \tag{1.1}$$

where θ is the constant axionic angle. This means that the constant value of the complex coupling constant, $\tau \equiv \tau_1 + i\tau_2 = C^{(0)} + ie^{-\phi}$, in the $AdS_5 \times S^5$ background is identified with the complex Yang-Mills coupling,

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g_{_{YM}}^2}\,. \tag{1.2}$$

In the following, τ will always be assumed to be equal to this constant value (mostly with $\tau_2^{-1} \ll 1$).

According to the prescription of [2, 3] the amplitudes of the bulk superstring theory in the $AdS_5 \times S^5$ background with fields propagating to specified values at points on the boundary are equivalent to correlation functions of composite operators in the boundary Yang-Mills theory. The boundary values of the bulk fields are interpreted as sources coupling to the operators in the Yang-Mills theory. Collectively denoting the independent cross ratios of the positions of the boundary fields by η, the resulting amplitude for an n-point function in the gauge theory can be written as a finite sum over contributions of the form

$$\mathcal{H}_n^s\left(\frac{\alpha'}{L^2},\tau,\bar\tau,\eta\right)A_n=\mathcal{H}_n^{YM}(\lambda,N,\theta,\eta)A_n\,. \tag{1.3}$$

The right-hand side is just a rewriting in terms of the variables N, λ and θ in which it is natural to express the correlation functions of the Yang-Mills theory. The finite sum involves factors A_n which span an independent set of tensor structures consistent with the space-time quantum numbers and symmetries of the n-point functions. Moreover, the A_n's all have a common factor, also dictated by symmetry (a function of the space-time separations $|x_i-x_j|$), which carries the dimension of the correlation function. Therefore, almost all the non-trivial information about the correlation functions really lie in the functions $\mathcal{H}_n$.

In the following, we will be exclusively concerned with the large-N limit of (1.3). The expansion of gauge theory amplitudes in $1/N$ translates into a small g expansion in the left-hand side of (1.3). The 't Hooft expansion of Yang-Mills amplitudes takes the familiar form (for convenience, we will drop the subscript n in much that follows),

$$\mathcal{H}^{YM}(\lambda,N,\theta,\eta)=N^2\left[\mathcal{H}_0^{YM}(\lambda,\eta)+\frac{1}{N^2}\mathcal{H}_1^{YM}(\lambda,\eta)+\cdots+\frac{1}{N^{2k}}\mathcal{H}_k^{YM}(\lambda,\eta)+\cdots\right]+\cdots \tag{1.4}$$

Here the second ellipsis includes Yang-Mills instanton terms of the form $e^{-2\pi|K|\frac{N}{\lambda}+iK\theta}$, each coming with its series of fluctuations. Although these instanton terms are exponentially suppressed they are uniquely specified by their phase. It is the structure of the 't Hooft expansion of fluctuations around a particular instanton background that we will focus on in section 2.

In the correspondence with the dual string theory, the terms in (1.4) which are powers of $1/N$ arise from perturbative string contributions with k being the world sheet genus. The instanton terms which are suppressed by powers of e^{-N} are non-perturbative and can be identified with D-instanton contributions [8]. Though the form of the $1/N$ expansion in (1.4) was originally motivated by weakly coupled perturbation theory, the existence of the dual string theory with the identifications (1.1) implies such a form should apply for all λ. In particular, we will exploit the existence of a well defined expansion for large λ that is defined by the α' expansion of the string theory.

2. Instanton non-renormalisation theorems

The instanton calculations in [6, 7] involved correlation functions of various combinations of the superconformal currents that make up a short (256 component) $\mathcal{N}=4$ supermultiplet. However, the general structure of interest to us does not depend on which of these correlation functions is considered so we will focus on a specific tensor structure in the correlation function of four stress tensors.

This particular tensor structure can be defined by its relation, via the AdS/CFT correspondence, to the $\mathcal{R}^4$ term in the type-IIB effective action (where $\mathcal{R}$ denotes the Weyl curvature). This ten-dimensional term has the tensor structure

$$\mathcal{R}^4 \equiv t^{M_1\cdots M_8} t_{N_1\cdots N_8} R^{N_1 N_2}_{M_1 M_2} \cdots R^{N_7 N_8}_{M_7 M_8}\,, \tag{2.1}$$

with t being a standard eighth-rank tensor. We wish to consider the linearization of the four curvatures around the $AdS_5 \times S^5$ background, keeping only the polarisations in the AdS_5 directions. The four-graviton scattering amplitude is expressed as a functional of the boundary (S^4) values of the graviton by attaching a spin-two bulk-to-boundary propagator to each graviton leg in the linearized vertex. Since the boundary graviton field is interpreted in the Yang-Mills theory as the source for the stress tensor, this procedure defines a particular tensor contribution to the correlation function of four stress tensors [5] which can be expressed as

$$\frac{L^2}{\alpha'} \tau_2^{1/2} f_1^{(0,0)}(\tau,\bar\tau) g_1(\eta) A_4\,. \tag{2.2}$$

The ${\alpha'}^{-1}$ dependence reflects the fact that the $\mathcal{R}^4$ term is of order ${\alpha'}^3$ relative to the Einstein–Hilbert term. The function A_4 has, in addition to the particular tensor structure determined by the bulk-boundary correspondence described above, a factor of $\prod_{i<j} |x_i - x_j|^{-8/3}$, which is fixed by conformal invariance. The residual dependence on the positions of the boundary operators is contained in the function of the two independent cross-ratios which has been denoted as $g_1(\eta)$. Explicit expressions for $g_1(\eta)$ in the case of closely related four-point functions were obtained in [6, 9]. The modular invariant function of the (complex) string coupling, $f_1^{(0,0)}(\tau,\bar\tau)$, is a nonholomorphic Eisenstein series that has the Fourier expansion in powers of $e^{2\pi i \tau_1}$ [8],

$$\begin{aligned} f_1^{(0,0)}(\tau,\bar\tau) &\equiv \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+n\tau|^3} = \sum_{K=-\infty}^{\infty} \mathcal{F}_K^1(\tau_2) e^{2\pi i K \tau_1} \\ &= 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2} + 4\pi \sum_{K=1}^{\infty} |K|^{1/2} \mu(K,1) \times \\ &\quad \times \left(e^{2\pi i K\tau} + e^{-2\pi i K\bar\tau}\right)\left(1 + \sum_{k=1}^{\infty} (4\pi K\tau_2)^{-k} \frac{\Gamma(k-1/2)}{\Gamma(-k-1/2)k!}\right). \end{aligned} \tag{2.3}$$

Here $\mu(K,1) = \sum_{d|K} d^{-2}$. The $K=0$ term contains the perturbative tree-level and one-loop contributions while the $K \neq 0$ terms are D-instanton contributions. The leading τ_2 independent term in the charge-K D-instanton sector was found to agree with a weak coupling Yang-Mills calculation in [7] (at least for the related sixteen-dilatino correlation function).

More generally, in the $AdS_5 \times S^5$ background there will be contributions proportional to A_4 which are of higher order in α'. These come from higher derivative terms in the effective action and can be studied in a Taylor expansion for small α'/L^2,

$$\mathcal{H}^s\left(\frac{\alpha'}{L^2}, \tau, \bar\tau, \eta\right) A_4 = \sum_{l=1}\left(\frac{\alpha'}{L^2}\right)^{l-2} F_l(\eta, \tau, \bar\tau)\, A_4\,, \tag{2.4}$$

where the $\mathcal{R}^4$ contribution is the first ($l = 1$) term in the series (so that $F_1(\eta, \tau, \bar\tau) = g_1(\eta)\tau_2^{1/2} f_1^{(0,0)}(\tau, \bar\tau)$). Examples of higher derivative terms in the ten dimensional effective action that would contribute to (2.4) include terms of the general form (in string frame)

$$(\alpha')^{2k-3}\int d^{10}x\, \sqrt{G^{(10)}}\, e^{(5k-11/2)\phi}\, F_5^{4k-4}\, \mathcal{R}^4\, f_k^{(0,0)}(\tau, \bar\tau)\,. \tag{2.5}$$

The modular functions $f_k^{(0,0)}$ that appear here, have been conjectured to be generalised Eisenstein functions [10, 11]. In the $AdS_5 \times S^5$ background with its constant five-form field strength F_5, these terms can give contributions proportional to A_4. Another class of terms suggested in [12, 13] involve derivatives acting on $\mathcal{R}^4$, which may also give nonzero contributions $\propto A_4$ in the $AdS_5 \times S^5$ background. There might also be terms that contribute to A_4 that cannot be expressed in terms of a local ten-dimensional action.

What about the possibility of terms which are non-perturbative in α', which are exponentially suppressed in the Taylor expansion, such as $e^{-L^2/\alpha'}$? For small α'/L^2, we understand terms of this type as coming from non-trivial saddle points of the world sheet theory, namely world-sheet instantons. But there are no topologically non-trivial two-cycles for the world-sheet to wrap in $AdS_5 \times S^5$ so such terms cannot appear in the string genus expansion.[1] One might argue that this does not rule out $e^{-L^2/\alpha'}$ contributions associated with non-perturbative τ dependence. But this also seems unlikely since we understand such non-perturbative terms as coming from D-instantons and again there is no obvious origin for world-sheet instanton contributions in a D-instanton background. We will therefore make the ansatz that (2.4) is the complete expression for the coefficient of A_4 in the correlation function of four energy-momentum tensors, at least for sufficently small α'/L^2. In practice, we will only need the weaker assumption that this is so for the D-instanton contributions to (2.4) (the terms with phases $e^{2\pi i K\tau_1}$, $|K| \geq 1$). Our arguments will show that this assumption is, at least, self consistent.

The $\mathrm{SL}(2,\mathbb{Z})$ duality symmetry of the IIB theory is related via the AdS/CFT correspondence to the Montonen-Olive duality of $\mathcal{N} = 4$ supersymmetric Yang-Mills

[1] In other instances of large-N gauge theories dual to closed strings, world sheet instantons are present and play an important role [14].

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theory. This requires that $F_l(\tau,\bar{\tau},\eta)$ has specific modular properties which means that it has the form,

$$F_l(\tau,\bar{\tau},\eta) = \tau_2^{1-l/2} H_l(\tau,\bar{\tau},\eta)\,, \tag{2.6}$$

where $H_l(\tau,\bar{\tau},\eta)$ is modular invariant (a scalar under $\mathrm{SL}(2,\mathbb{Z})$). The explicit power of $\tau_2^{1-l/2}$ arises from the transformation in the effective action from the string to the Einstein frame (where the metric is $\mathrm{SL}(2,\mathbb{Z})$ neutral) due to the factors of the metric appearing with the powers of α'/L^2. In terms of Yang-Mills variables, this is easy to see since the coefficient of H_l in (2.4) is the combination (from (2.4) and (2.6))

$$\left(\frac{\alpha'}{L^2}\right)^{l-2} \tau_2^{1-l/2} = \frac{1}{\lambda^{l/2-1}} g_s^{l/2-1} = N^{-l/2+1}\,, \tag{2.7}$$

which is inert under $\mathrm{SL}(2,\mathbb{Z})$.

Now consider the non-perturbative part of the modular function $H_l(\tau,\bar{\tau},\eta)$ coming from BPS charge-K D-instantons. This amounts to picking out the saddle point (when $g = \tau_2^{-1} \ll 1$) with the exponential $e^{2\pi(iK\tau_1 - |K|\tau_2)}$ dependence.[2] We expect this to take the generic form,

$$H_l(\tau,\bar{\tau},\eta)|_K = d(K,l) e^{-2\pi(|K|\tau_2 - iK\tau_1)} \left[h_0^{(l)}(\eta) + {\tau_2}^{-1} h_1^{(l)}(\eta) + {\tau_2}^{-2} h_2^{(l)}(\eta) + \cdots\right], \tag{2.8}$$

an expression which deserves further explanation. Firstly, the successive terms in this series are spaced in integer powers of ${\tau_2}^{-1}$ since they arise in string theory from world-sheet configurations with increasing numbers of boundaries with Dirichlet conditions and/or handles. The functions $h_i^{(l)}(\eta)$ (which also depend on the instanton charge K), that appear here, are severely constrained by the fact that H_l is a modular function. Secondly, we have assumed that there is no $\tau_2 = g^{-1}$ dependence in the overall factor $d(K,l)$. Any such dependence would have to be a power that arises from the zero mode integrations around the D-instanton. This power should not depend on l. But we know from the $l = 1$ case (2.3) that there is no such overall factor.[3] This statement should also be a consequence of supersymmetric cancellations of bosonic zero mode contributions with fermionic ones. Therefore, this might be special to a class of correlation functions such as the ones in the short multiplet we are concerned with.

Rewriting the amplitude in terms of N, λ and θ and summing the total contribution to the K instanton background from all powers in the derivative expansion

[2]In the charge-K sector with phase $e^{2\pi iK\tau_1}$ there could be contributions from non-BPS configurations of $K+K'$ instantons and K' anti-instantons. These would be suppressed by an additional factor of $e^{-4\pi K'N/\lambda}$. Due to their different N dependence such terms would not enter our considerations, even if they were present.

[3]Actually, the essence of our conclusion will not be affected even if we had an overall factor of g^{n_l}, with n_l taking values over non-negative integers.

(all l) gives (using (2.4), (2.6) and (2.8))

$$\mathcal{H}^{YM}|_K = e^{-2\pi\frac{|K|N}{\lambda}+iK\theta}\sum_{l=1} d(K,l)N^{1-l/2}\times$$
$$\times\left[h_0^{(l)}(\eta)+\left(\frac{\lambda}{N}\right)h_1^{(l)}(\eta)+\left(\frac{\lambda}{N}\right)^2 h_2^{(l)}(\eta)+\cdots\right]. \tag{2.9}$$

This can be reorganized into a 't Hooft expansion by grouping together the powers of $1/N$, which gives the perturbation expansion around the BPS K-instanton configuration,

$$\mathcal{H}^{YM}|_K = e^{-2\pi\frac{|K|N}{\lambda}+iK\theta}\sum_{m=1} N^{1-m/2} f_m(\lambda,\eta)\,. \tag{2.10}$$

At first sight it might seem surprising that the series of fluctuations about an instanton should have a 't Hooft expansion in powers of $N^{-1/2}$. From the string viewpoint the spacing by half-integer powers of $1/N$ arises very naturally from the presence of all integer powers of α' in (2.4). Furthermore, this feature is confirmed directly in the Yang-Mills theory by a saddle point analysis of the contribution of the exact zero-mode measure (as for example in [7, eq. (5.7)]). For general instanton number $K>1$ the fluctuations around the saddle point are in powers of $N^{-1/2}$. The series of fractional powers of N therefore arises from the K-instanton measure. Only in the case $K=1$ does the series consist of terms with integer spaced powers of $1/N$ (starting with $N^{1/2}$).

The key point following from the structure of (2.9) is that the power of λ in the expansion is always bounded by that of N. In other words, each function $f_m(\lambda,\eta)$ in (2.10) is a polynomial in λ,

$$f_m(\lambda,\eta) = \sum_{k=0}^{[\frac{m-1}{2}]} \lambda^k h_k^{(m-2k)}(\eta). \tag{2.11}$$

For example,

$$f_1(\lambda,\eta) = h_0^{(1)}(\eta)\,, \qquad f_2(\lambda,\eta) = h_0^{(2)}(\eta)\,, \qquad f_3(\lambda,\eta) = h_0^{(3)}(\eta)+\lambda h_1^{(1)}(\eta)\,,\ldots \tag{2.12}$$

By our initial arguments, this is the complete form of the answer for small α' or equivalently large λ. Barring the (unlikely) possibility of a phase transition as a function of λ, the knowledge that f_m is a polynomial in positive powers of λ at large λ allows it to be analytically continued to weak 't Hooft coupling.[4] But then the polynomial form of f_m means that there are only a finite number of terms in the small-λ perturbation expansion for each power of N.

[4]Even though the orginal expansion in α' (2.4) and g (2.8) may only be asymptotic, we can nevertheless trust (2.11) to small λ. This is similar to the statement that though the full perturbative expansion in gauge theories has zero radius of convergence, the planar diagram expansion can be trusted in some finite radius [15].

Thus the very structure of the string expansion implies a sequence of non-trivial non-renormalisation theorems for 't Hooft perturbation theory around an instanton background. In particular, the leading large-N term comes from $m = 1$ and that is just a constant as far as its dependence on λ is concerned. In other words, it receives only a semi-classical contribution. This 'explains' the fact that the semi-classical approximation to the K-instanton contribution to $\mathcal{N} = 4$ SU(N) Yang-Mills theory at leading order in N (the term of order $N^{1/2}$ which was evaluated in [7]) agrees precisely with the expression predicted by the AdS/CFT correspondence [5]. Moreover, from (2.12) we see that the next to leading term should behave as $N^0 h_0^{(2)}(\eta)$ and is also independent of λ and therefore semi-classically exact. Verifying this prediction would require knowledge of the $l = 2$ term in (2.4) that contributes at order $(\alpha')^0$.

Other predictions can also be tested. For instance, the structure of f_3 in (2.12) requires that the next term in the $1/N$ expansion has only two terms in the loop expansion. Also, in the special case of instanton number $K = 1$ the semi-classical contribution was computed in [16] for all values of N. In this case the expansion is in integer powers of $1/N$, which immediately determines that

$$h_0^{(2k)}(\eta, K=1) = 0\,, \qquad h_0^{(2k-1)}(\eta, K=1) = b_k g_1(\eta) \tag{2.13}$$

where b_k are the coefficients in the exact answer [16],

$$\frac{\Gamma(N-1/2)}{\Gamma(N-1)} = N^{1/2}\sum_{l=1}^{\infty} b_k N^{-k+1} = N^{1/2}\left(1 - \frac{5}{8}\frac{1}{N} - \frac{23}{128}\frac{1}{N^2} + \cdots\right). \tag{2.14}$$

3. Comments and conclusions

We have seen by making rather minimal assumptions that certain instanton contributions in the 't Hooft limit of $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory receive only a finite number of perturbative corrections at a given order in the $1/N$ expansion. More precisely, at order $N^{1-m/2}$ there are $[\frac{m+1}{2}]$ terms in the power series in the 't Hooft coupling, λ, starting with λ^0. It should be emphasised that, unlike with usual non-renormalisation theorems, our statements only apply at each order in $1/N$ in the 't Hooft limit whereas for finite N perturbative terms appear at all loops. Although our arguments do not make direct use of supersymmetry, this enters indirectly since the AdS/CFT correspondence does require supersymmetry. This is similar in spirit to the way in which the mere existence of a Lorentz invariant eleven-dimensional limit of M-theory implies non-trivial facts about D0-brane quantum mechanics, as in the DLCQ description.

The assumptions we have made have the virtue that they can be checked by direct evaluation of perturbative contributions in the large-N expansion of the gauge theory. In this way one could investigate to what extent these results apply to

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theories with less supersymmetry. It would, for instance, be interesting to find out how much can be said about the conformal field field theories described in [17, 18]. In such cases the dual string theory has an S^5/Γ sector which could admit world sheet instantons whose absence was one of the important ingredients in our argument.

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